Estimating Monthly Reference Evapotranspiration In Najran Region, KSA, Using S (REG_AR) Model

Ahmed Helmy, and Mohamed A. Dahim

***Abstract*—** High percent of the Arab countries suffer from great shortage in their water resources because it lies in hot arid and semiarid regions. However, vast areas of these regions are utilized for agricultural production, which require making essential studies regarding the determination of water requirements for irrigation purposes, i.e. determination of crops' consumptive use of water in such areas. Crops' consumptive use of water is related to reference evapo-transpiration by a coefficient called crop coefficient, which depends on type of crop and its growing stage. In addition, evaporation measurements are of great importance for irrigation systems design, irrigation rotations, and development of hydrologic models. In the present research, a seasonal regression-autoregressive *S(REG_AR)* model has been developed and used to simulate and estimate monthly reference evapo-transpiration in Najran Region, KSA. Actually, the present research is a part of a two-year research project, which has been planned and funded by Najran University, KSA. The performance of the developed model has been evaluated by calculating the statistical characteristics of both the estimated and measured values of monthly evapo-transpiration in each of the historical and validation periods. Analysis of results revealed the adequacy of the developed model and hence its capability of doing good estimation of monthly evapo-transpiration in Najran Region, KSA and in regions of similar climatic conditions.

***Keywords*—**Blaney - Criddle, Consumptive use of water, Dummy variables, Evapo-transpiration, "S(REG_AR)" model.

I. Introduction

IN the present time, the study of water resources evaluation for agriculture purposes became one of the basic and strategic tasks especially in hot arid regions that suffer from great shortage in their water resources. Several classical empirical methods and equations are in use up today in estimating consumptive of water for agricultural crops based on the estimation of the potential evapo-transpiration. The use of such empirical equations likes that of Turc, Blaney - Criddle, Thornthwaite, Penman and others did not take into consideration the hydro-energetic and agricultural characteristics of the arid regions [3]. Because these equations were derived for the use in humid and sub humid regions, their use in arid regions is highly affected by the acute climatic factors in such regions [9].

Evapo-transpiration is influenced by solar radiation, air temperature, vapor pressure, wind, and (minimally by atmospheric pressure) [6]. It is also varies with latitude, season, time of day, and sky condition [8], [13]. Evapo-transpiration reflect changes in the moisture deficiency of a basin and are sometimes used to estimate stream runoff for river forecasts. Estimates of these factors are also used in determining water supply requirements for proposed irrigation projects [10].

Several authors have developed several stochastic generation procedures for evaporation and evapo-transpiration sequences. Examples of such procedures include the work of [2], [15]. In many situations, it is advantageous to calculate rather than measure evapo-transpiration using expensive field measuring equipment. Consumptive use of water is related to the estimated evaporation by some coefficient, which depends on the type of the crop planted and its stage of growing [3].

The present study aims to examine the possibility of developing an empirical equation that allows calculating the monthly reference evapo-transpiration in Najran Region, KSA. Actually, the present research is a part of a two-year research project, which has been planned and funded by Najran University, KSA.

II. Association Of Evaporation With Available Climatic Variables

The correlation relationship between evaporation-transpiration and some of the available climatic factors (e.g. a change in air temperature, the number of daylight hours, wind speeds, relative humidity. etc) can be studied. Data on these variables is available throughout the year in most of the climate monitoring stations in the arid regions [6], [11]. The correlation coefficient is one of the common ways to indicate a correlation between two variables or not. The plot of the cross correlation function *CCF* between two variables is also one of the most accurate methods to determine whether there was a correlation between these two variables [1].

III. Development Of *S(Reg-Ar)* Model

Regression analysis is based on the relationship, or association, between two or more variables. The known variable (or variables) is called the independent variable(s).

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The variable we are trying to estimate is the dependent variable.

The most glaring problem revealed by fitting ordinary regression models to hydrologic time series is the autocorrelation of the residuals (the difference between the measured and estimated values from the regression model). Auto correlated residuals commonly occur when omitting important explanatory variables from the regression analysis. When the residuals from a regression model are strongly auto correlated, one cannot rely on the results. The significant levels reported for the regression coefficients are wrong and the other statistical measures that measure the accuracy of the regression model does not accurately summarize the explanatory power of the independent variables. The seasonal Regression Autoregressive *S(REG_AR)* model corrects this problem and gives estimates that are more reliable. It estimates true regression coefficients from time series that has some order of auto-correlated errors [1].

The various steps in the modeling process with the *S(REG_AR)* model are summarizes as follows:

Step 1. Building a seasonal regression model to the dependent variable (evapo-transpiration) based on number of available climatic variables as independent variables (air temperature, solar radiation, sunlight ours, relative humidity, wind speed, etc.) which are easy to measure and are strongly correlated with evapo-transpiration.

One way to include seasonal effects in a regression model is to use dummy variables for the seasons. A dummy variable is that equals 1 for observations in a specific month and is undefined (technically missing) for observations not in that month. Each of these dummy variables is equivalent to a new independent variable. When choosing the number of dummy variables, the rule is to use (*P-l*) dummy variables to denote (*P*) different periods [12]. For monthly data (*P=12*), we therefore use only 11 dummy variables.

The use of regression analysis in the determination of evaporation (*E*) as a dependent variable with a number *n* of climatic variables and *11* dummy variables as independent variables in the model can be expressed as:

$$E_{o,m} = C_o + \sum_{1}^{n} C_n . V_{n,m} + \sum_{m=2}^{12} S_m . D_m + E_{e,m} \qquad (1)$$

Where:

$E_{o,m}$ is the value of Evapo-transpiration in month *m*,

$V_{n,m}$ is the value of n umber of climatic variables in month *m* that are strongly correlated with $E_{o,m}$,

D_m is a dummy variable of value *1* for month *m* and *zero* otherwise,

C_o, C_n, S_m are respective constants in the regression model, the value of which can be estimated from regression analysis, and

$E_{e,m}$, is the error left in month m after fitting the seasonal regression model to $E_{o,m}$.

An estimate of $E_{o,m}$ is $\hat{E}_{o,m}$ such as:

$$\hat{E}_{o,m} = C_o + \sum_{1}^{n} C_n . V_{n,m} + \sum_{m=2}^{12} S_m . D_m \qquad (2)$$

An estimate of the error series $E_{e,m}$ is such as:

$$E_{e,m} = E_{o,m} - \hat{E}_{o,m} = E_{o,m} - \left(C_o \sum_{1}^{n} C_n . V_{n,m} + \sum_{m=2}^{12} S_m . D_m\right) \qquad (3)$$

The error series "(3)" is examined by the aid of the autocorrelation function *ACF* and the partial autocorrelation function *PACF* plots to see if it is auto-correlated or not [4], [5], [7]. If $E_{e,m}$ is not correlated, the *S(REG_AR)* model is simplified to *S(REG)* model and then use (1) to estimate evapo-transpiration. If $E_{e,m}$ is auto-correlated, move to *step2*.

Step2. Specifying the seasonal autoregressive *SAR* model of order *p, P* for the error series $E_{e,m}$ where *p* is the non-seasonal order and *P* is the seasonal order. The value of *p, P* can be identified by the aid *ACF* and *PACF* of the error series. An *SAR* model of *p=3* and *P=2* is always convenient to estimate the error series of seasonal behavior.

The estimated *SAR* model of order *p=3, P=2* can be expressed as:

$$\hat{E}_{e,m} = \sum_{p=1}^{3} \emptyset_p . E_{e,m-p} + \sum_{P=1}^{2} \Phi_P . E_{e,m-12P} - \sum_{p=1}^{p=3} \sum_{P=1}^{P=2} \emptyset_p . \Phi_P . E_{e,m-p-12P} + \epsilon_{e,m} \qquad (4)$$

Where:

$\hat{E}_{e,m}$ is the estimate value of $E_{e,m}$,

$\emptyset_p$, Φ_P is the value of non-seasonal and seasonal coefficients of order *p, P* respectively, and

$\epsilon_{e,m}$ is the error left in month m after fitting the seasonal regression model to $E_{e,m}$.

Step3. The seasonal regressive *SREG* model and the seasonal autoregressive *SAR* model, "(2)" + "(4)", are combined to obtain the whole *S(REG_AR)* model as:

$$\hat{E}_m = \hat{E}_{o,m} + \hat{E}_{e,m} = C_o + \sum_{1}^{n} C_n . V_{n,m} + \sum_{m=2}^{12} S_m . D_m + \sum_{p=1}^{3} \emptyset_p . E_{e,m-p} + \sum_{P=1}^{2} \Phi_P . E_{e,m-12P} - \sum_{p=1}^{p=3} \sum_{P=1}^{P=2} \emptyset_p . \Phi_P . E_{e,m-p-12P} \qquad (5)$$

The general Equation of the *S(REG_AR)* after expanding and rearranging it terms can be expressed as:

$$\hat{E}_m = C_m + \sum_{1}^{n} C_n . V_{n,m} + \sum_{p=1}^{3} \emptyset_p . E_{net,m-p} + \sum_{P=1}^{2} \Phi_P . E_{net,m-12P} - \sum_{p=1}^{p=3} \sum_{P=1}^{P=2} \emptyset_p . \Phi_P . E_{e,m-p-12P} \qquad 6)$$

Where:

$$E_{net,m} = E_m - C_m - \sum_{1}^{n} C_n . V_{n,m} \qquad (7)$$

"Cm" is a constant that varies with month and depends on the seasonal behavior of the series. It can be expressed as:

$$C_m = S_m + C_0\left(1 - \sum_{p=1}^{3} \emptyset_p - \sum_{P=1}^{2} \Phi_P + \sum_{p=1}^{p=3} \sum_{P=1}^{P=2} \emptyset_p . \Phi_P\right) \qquad (8)$$

Several leading indicators can be introduced in the model (if available) such as relative humidity or any other climatic factors influencing evaporation.

Step4. Diagnostic checking:

After having estimated the parameters of the final *S(REG_AR)* model, it is necessary to do diagnostic checking to verify that the chosen model is adequate. There are two ways of doing this:

1. Studying the residuals $\in_{e,m}$ to see if any pattern remains unaccounted for. The plot of the *ACF* of $\in_{e,m}$ can do this. These residuals are hopefully just random noise if we find no significant autocorrelations.
2. Studying the sampling statistics (for both $E_{o,m}$ and $\hat{E}_{o,m}$) to check the performance of the developed model.

Step5. Using the developed model, an estimation of the monthly evaporation can be easily made.

IV. Case Study

The pre-outlined steps of developing the *S(REG_AR)* model are applied and used to simulate and estimate the evaporation - transpiration in the study area that fall within Najran region, KSA.

Najran is located in the southwestern part of the Kingdom of Saudi Arabia and is located between 17^0 to 20^0 degrees north and between 44^0 to 52^0 in the east, with an area of about 365 thousand km^2 including El Kherkhir area. Najran Region is characterized by a mild climate in winter with medium moisture and freezing in the mountainous areas, and moderate temperature in summer. The rainfall is moderate in winter and often heavy in summer. Temperatures vary between about 15 and even 30 degrees Celsius. Data regarding different hydro-climatic parameters are collected on daily basis from Najran hydro-meteorological station located in Najran Airport. The meteorological station of Najran, serial number (St. No. 41128) is located between at Latitude 17^0 $36^{\backslash}$ $41^{\backslash\backslash}$ North and Longitude 44^0 $24^{\backslash}$ $49^{\backslash\backslash}$ in the East and at an elevation of 1212.33 m above mean sea level. The data collected include daily maximum and minimum air temperatures, daily maximum and minimum relative humidity, daily maximum wind speed, daily number of sun hours, daily solar radiation, daily rain fall, and daily measured potential evapo-transpiration for the period from January 2006 and up to October 2013. The collected daily data are statistically examined to investigate its homogeneity. The correlation relationships between the different hydro-climatic variables are examined. By the aid of correlation relations between the collected data, empirical equations are developed and used to estimate the missed values in the climatic variables. The final daily data set are summarized to monthly data.

In this analysis, the monthly data for the period 2006-2008 are taken as a historical period for calculating the model parameters while the data for year 2009 are taken as validation period for checking the performance of the developed model.

A. Association of Evapo-transpiration with Available Climatic Factors in the Study Area

Figs.1 (a), (b), and (c) show the plot of the cross correlation function *CCF* between mean monthly evapo-transpiration *E* with each of mean monthly temperature (*T*), mean monthly solar radiation (*SR*), and mean monthly relative humidity(*RH*) respectively. A significant correlation exists between *E* and *SR*. One can introduce solar radiation *SR* in the model as independent variables when estimating *E* in the study area.

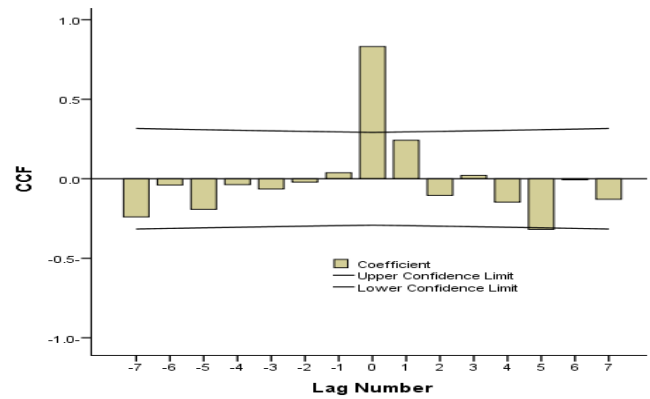

Fig. 1(a) Cross Correlation between E & SR

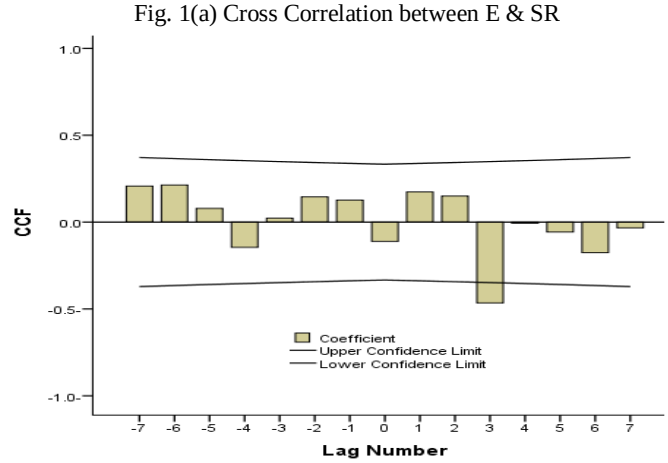

Fig. 1(b) Cross Correlation between E & Mean Temperature

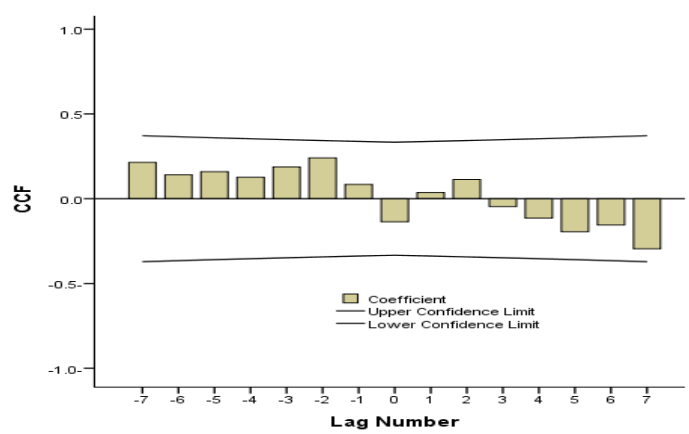

Fig. 1(c) Cross Correlation between E & Mean RH

B. The Developed Model

First Step:

The following equation expresses the first part of the model *SREG*, "(2)", after omitting non-significant dummy variables [14]:

$$\hat{E}_{o,m} = 0.303SR + 0.23V_{2,m} + 0.47V_{3,m} + 0.64V_{4,m} + 0.8V_{5,m} + 0.78V_{6,m} + 0.87V_{7,m} + 0.9V_{8,m} + 0.93V_{9,m} + 0.36V_{10,m} + 0.43V_{11,m} \quad (9)$$

Equation (9) can be summarized to the form:

$$\hat{E}_{o,m} = 0.303SR + S_m \quad (10)$$

Where S_m are seasonal factors and varies according to month (*M*) as presented in *Table (1)*. Referring t "(8)", $S_0 = 0$, and hence $C_m = S_m$.

Fig. (2) shows the plot of the *ACF* of the error series $E_{e,m}$ left after fitting *SREG* model.

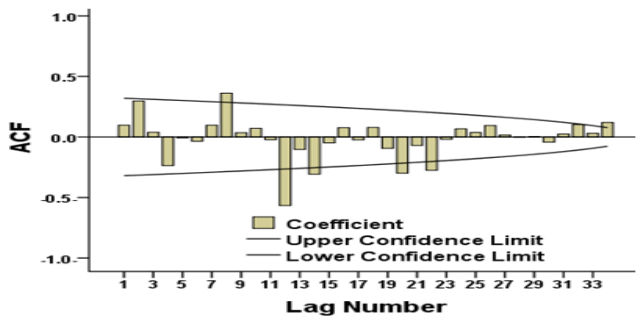

Fig. (2) ACF of the Error Series ($E_{e,m}$) Left After Fitting *SREG* Model

The plot indicates a significant correlation at lag12. At such case, for simplicity , it is possible to ignore such significant correlation and the *S(REG_AR)* model may be simplified to *S(REG)* model and then use "(10)" to estimate evapo-transpiration and move directly to step4, diagnostic checking, to examine the performance of the *SREG* model. Otherwise, the error series $E_{e,m}$ may be estimated by an *SAR* model of the order *p=0, P=1* [1] and move to *step2* of modeling procedure.

Second Step:

The developed *SAR* model is expressed by the following equation:

$$\hat{E}_{e,m} = -0.645\, E_{e,m-12} + \epsilon_{e,m} \quad (11)$$

Third Step:

The two models *SREG* and *SAR* are combined to obtain the total *S(REG_AR)* model after expanding and rearranging its terms as:

$$\hat{E}_m = 1.645S_m + 0.303SR_m + 0.2SR_{m-12} - 0.645E_{m-12} + \epsilon_{e,m} \quad (12)$$

Fourth Step:

Model Verification by:

1. Studying the final residuals $\epsilon_{e,m}$*:*

Fig.3 shows the plot of the *ACF* of $\epsilon_{e,m}$ after fitting *S(REG_AR)* model. No significant correlations appear at seasonal lags which indicates that the final errors are random and hence the model is more adequate than the *SREG* model, *Fig.2*.

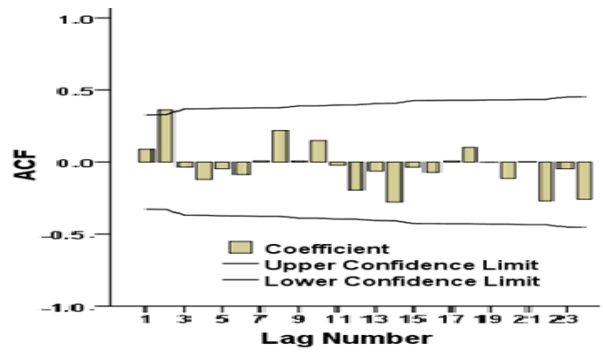

Fig. 3 ACF of $\epsilon_{e,m}$

2. Studying the sampling statistics (for both $E_{o,m}$ *and* $\hat{E}_{o,m}$*):*

Table II indicates the statistical characteristics (mean, sum, and standard deviation) of $E_{o,m}$ and $\hat{E}_{o,m}$ in the historical period (2006-2008) using both *SREG* and *S(REG _AR)* models.

The statistical measures shown indicate that both the *SREG* and *S(REG_AR)* models adequately fit the evapo-transpiration in the historical period.

3- *Graphical plot of both* $E_{o,m}$ *and* $\hat{E}_{o,m}$*:*

*Figs.4*a, and 4b show the plot of $E_{o,m}$ and $\hat{E}_{o,m}$ in the historical period (2006-2008) using both *SREG* and *S(REG _AR)* models respectively. As shown from the figure, the measured and the estimated values of evapo-transpiration are more close to each other when using S(REG_R) model than that when using the SRG model and both indicate the accuracy of the developed models.

Fifth Step:

The developed models, "(10)", "(12)" are used to estimate monthly evapo-transpiration in the validation period, year 2009.

"Table.III" indicates the statistical characteristics (mean, sum, and standard deviation) of "$E_{o,m}$" and "$\hat{E}_{o,m}$" in the validation period (2009) using both *SREG* and *S(REG _AR)* models. The results show also the accuracy of the developed models in the validation period.

Figs. 5 (a), and (b) show the plot of "$E_{o,m}$" and "$\hat{E}_{o,m}$" in the study area for the validation period, 2009 using both *SREG* and *S(REG _AR)* models respectively. The SREG model do good prediction of *E* than the *S(REG_AR)* model in the validation period and both can be used to estimate evapo-transpiration in the coming years based on the available climatic variables.

TABLE I
VALUES OF S_M ACCORDING TO MONTH (M)

M	1	2	3	4	5	6	7	8	9	10	11	12
S_m	0	.23	.47	.64	.8	.78	0.87	.9	.93	.36	.43	0

TABLE II
THE STATISTICAL CHARACTERISTICS OF BOTH ($E_{O,M}$) AND ($\hat{E}_{O,M}$) IN THE HISTORICAL PERIOD [14]

Period		Mean (mm)		Sum (mm)		Standard Deviation	
		$E_{o,m}$	$\hat{E}_{o,m}$	$E_{o,m}$	$\hat{E}_{o,m}$	$E_{o,m}$	$\hat{E}_{o,m}$
(2006-2008)	SREG	3.638	3.637	130.98	130.92	0.673	0.661
	S(REG _AR)		3.660		131.77		0.660

TABLE III
THE STATISTICAL CHARACTERISTICS OF BOTH ($E_{O,M}$) AND ($\hat{E}_{O,M}$) IN THE VALIDATION PERIOD [14]

Period		Mean (mm)		Sum (mm)		Standard Deviation	
		$E_{o,m}$	$\hat{E}_{o,m}$	$E_{o,m}$	$\hat{E}_{o,m}$	$E_{o,m}$	$\hat{E}_{o,m}$
(2009)	SREG	3.806	3.852	45.67	46.22	0.735	0.797
	S(REG _AR)		3.880		46.56		0.795

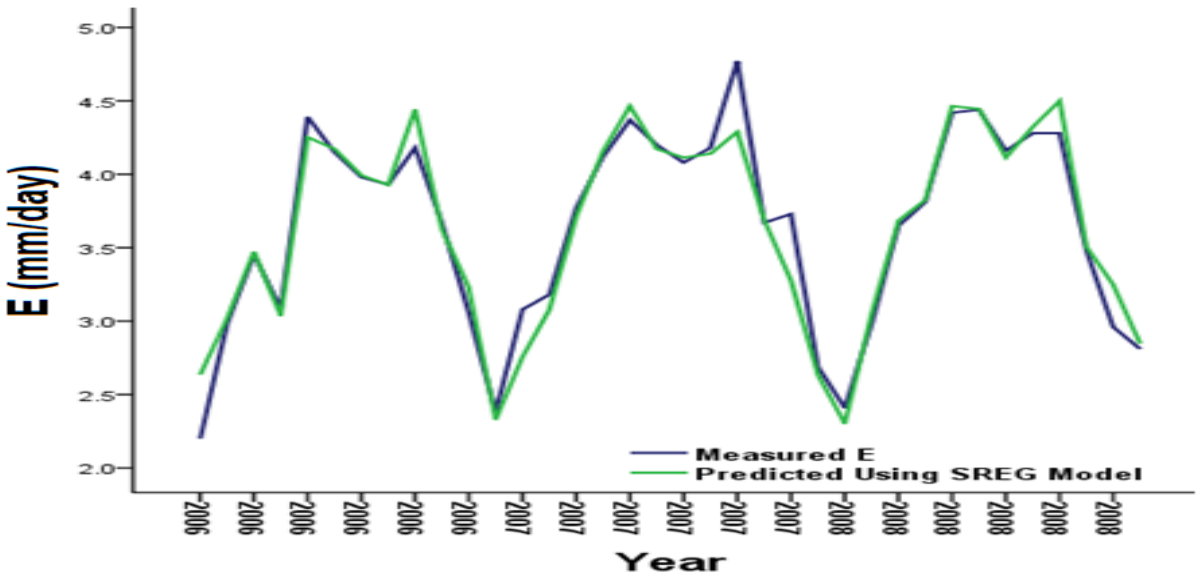

Fig. 4(a) Plot of both Measured and Calculated E in the Historical Period (2006-2008) Using SREG Model

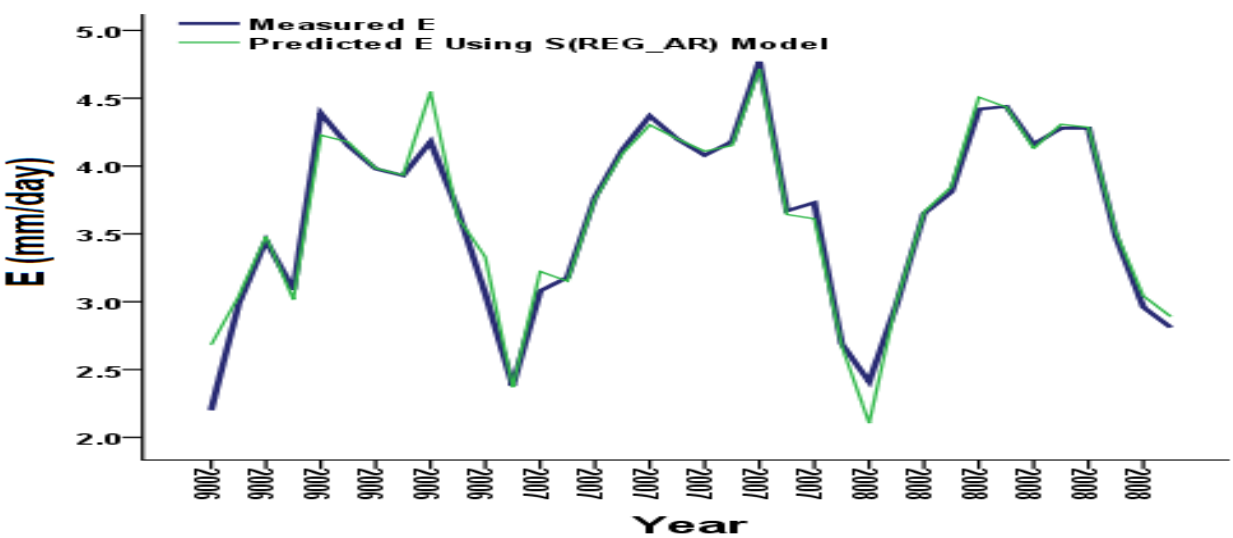

Fig. 4(b) Plot of both Measured and Calculated E in the Historical Period (2006-2008) Using S(REG_AR) Model

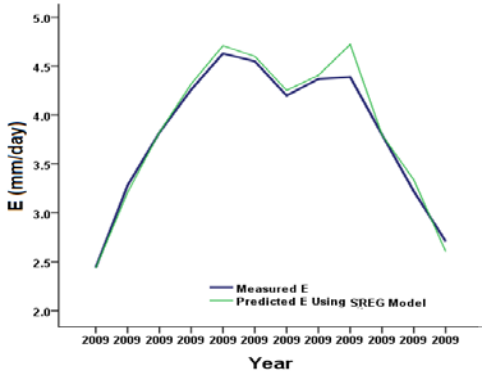

Fig. 5a:Plot of both Measured and Calculated E in the Validation Period (2009) Using SREG Model

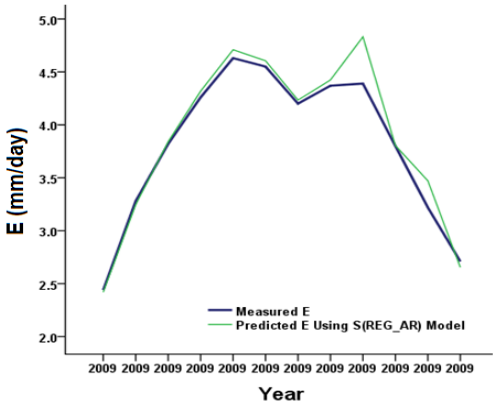

Fig. 5b:Plot of both Measured and Calculated E in the Validation Period (2009) Using S(REG_AR) Model

V. Summary And Conclusion

Association between monthly evapo-transpiration in the study area and available climatic factors such as mean monthly air temperature, solar radiation, and relative humidity is examined using the cross-correlation technique. Seasonal regression-autoregressive "S(REG_AR)" model is developed to estimate monthly evapo-transpiration in Najran Region, KSA, and the results permit the following conclusions:

1. There is a good association between evapo-transpiration and solar radiation. Influence of other climatic factors on evapo-transpiration should also be examined when available.
2. Diagnostic checks applied to the developed models revealed their adequacy and hence their capability of doing good estimation of monthly evapo-transpiration in the study area.

The methodology is applicable for any monthly hydrologic time series that is auto correlated.

3. The methodology should be, hopefully, useful for estimating rather than measuring, monthly evapo-transpiration in such arid regions that have the same climatic conditions.

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