Numerical Analysis of Direction of Arrival Estimation Using ESPRIT Algorithm

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Abstract— This paper introduces a study on angle of arrival (DOA) estimation using the ESPRIT algorithm. This technique utilizes principles grounded in subspace methodologies to successfully segregate the eigenvectors into two well-defined subspaces: one dedicated to the signal and the other to noise. The performance of the effectiveness of a direction of arrival (DOA) estimation algorithm based on subspace principles is carried out using a uniform linear array (ULA) configuration. The outcomes from the simulation highlight the critical significance of meticulously choosing parameters, such as sample and element numbers, signal-to-noise ratio (SNR), and antenna element spacing. These selections substantially influence the algorithms' ability to accurately estimate the direction of arrival for incoming signals.

Index Terms— Estimation of DOA, ESPRIT Algorithm, Uniform Linear Array

I. Introduction

Over the past decade, wireless communication systems have garnered significant global attention due to their capacity to enhance spectral efficiency and elevate performance within mobile communication networks [1]- [3]. The estimation of direction of arrival (DOA) poses as one of the most formidable challenges in localizing and tracking multiple moving sources, encompassing applications in radar, mobile communications, and various other fields. The development of antenna techniques used in communications systems quickly raised the problem of the clutter and the reliability of the used components [4]. Utilizing smart antenna arrays consisting of multiple antenna elements also enables the identification of spatial signal characteristics, such as determining the signal's direction of arrival (DOA). These antennas represent a highly auspicious technology for facilitating wireless communication systems with substantial capacity enhancements [5].

Moreover, the operating principle of smart antennas entails dynamically shaping a beam pattern, enabling adjustments to steer the main lobe toward the intended signal direction, while concurrently introducing nulls in the undesired signal direction, typically where interference is present [6]. The significance of direction of arrival (DOA) estimation lies in its pivotal role within smart antennas. By accurately estimating the DOAs of incoming signals, the system can substantially enhance its capacity and throughputs. Essentially, DOA estimation harnesses information received by the array's antenna elements to discern the directions of signals originating from users and interference sources. Once these DOA estimates are obtained, interference mitigation becomes feasible through null steering, which involves placing nulls in the detected direction of interference [7]. Numerous DOA algorithms have emerged over the last four decades, including methodologies like the Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT). Nonetheless, the subspace technique exhibit enhanced efficacy but entails greater computational intricacy. The ESPRIT algorithm, introduced initially by Roy and Kailath in 1989, exploit the rotational invariance inherent in the signal subspace formed by a pair of arrays exhibiting translational invariance. This method employs two identical arrays derived from the original array, each possessing a translational invariance configuration. The scope of this paper is to propose and estimate of direction of arrival DOA. Our suggested device is formed by using of ESPRIT algorithm. Section 2 briefly describes the mathematical aspects behind the algorithm. The outcomes of the simulation are elaborated upon in Section 3, with Section 4 encompassing the main conclusion drawn from our study.

II. Problem Formulation

The system of smart antenna array comprises multiple elements, uniformly spaced from one another. Among various types of smart antennas, the adaptive antenna array system distinguishes itself with its ability for adaptive beamforming and precise estimation of angles of arrival (DOA), illustrated in Figure 1. Essentially, this system is built upon an array of antennas, complex weights, a sum and a signal processing unit [8]. The core concept of the envisioned smart antenna system is formed by a signal processing unit dedicated to advancing efficient algorithms for both direction of arrival (DOA) estimation and adaptive beamforming techniques. The initial phase of the presented design commences with the calculation of the covariance matrix. Assuming the presence of L uncorrelated narrowband sources emitting plane wave signals, denoted as $S_L(t)$, from a defined angle θ, the incoming waves are captured by multiple elements, leading to variations in arrival times. As a result of this temporal disparity, the composite received signal $x_M(t)$, can be formulated as follows:

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$$x_M(t) = \sum_{i=1}^{L} a(\theta_L).s_L(t) + b_M(t) \qquad (1)$$

The formula (1) can be expressed in the following manner

$$x(t) = A(\theta)s(t) + b(t) \qquad (2)$$

Where

$x(t)$:is received signal

$b(t)$:is additive noise vector with component of variance σ^2.

s(t) :is the original signal from the L sources

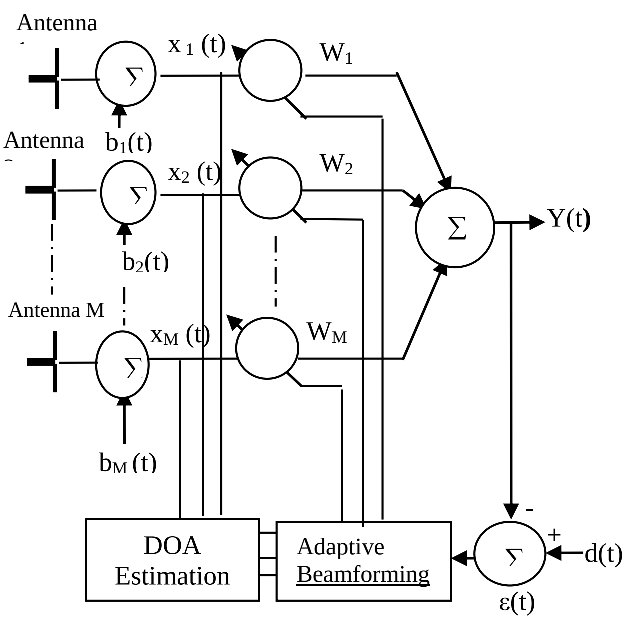

Fig. 1. Structure of a smart antenna system

Illustration 2 depicts an array containing M elements, each separated by a distance of d. Upon the arrival of an incident wave carrying a baseband signal s(t) at the antenna array, positioned at an angle θ_i, the response of the linear array, denoted as A(θ), to the source arriving from the direction θ_i (where i varies from 1 to L), can be determined through the resources mentioned in citations [9] and [10].

The presented challenge entails the estimation of the spatial covariance matrix, symbolized as R_{xx} , pertaining to the observed signal vector x(t) with the utilization of N signal samples through an array's output.

$$R_{xx} = E[x.x^H] = \frac{1}{N}\sum_{i=1}^{N} x(i)\,.x^H(i) \qquad (3)$$

It is within our assumption that the array has observed N samples, thereby giving rise to the array output vector represented as x(n)

$$x_M(n) = a(\theta).s(n) + b_M(n) \text{with} \quad n=1,..........N \qquad (4)$$

Where

N : is the number of signal samples or snapshots (N).

$(.)^H$:is hermitian conjugate transpose

In multiple x (t) by $x^H(t)$, the average of the equation gives

$$E[x(t).x^H(t)] = a(\theta).E[s(t).s^H(t)].a^H(\theta) + a(\theta).E[s(t).b^H(t)] + E[b(t).s^H(t)].a^H(\theta) + E[b(t).b^H(t)] \qquad (5)$$

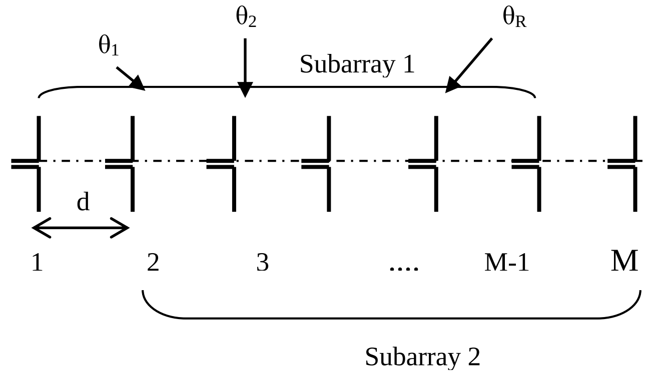

Fig. 2. Antenna array structure for ESPRIT algorithm

If we assume that $\bar{s}(t)$ and $\bar{b}(t)$ are decorrelated and statistically independent, then equation (5) becomes

$$R_{xx} = a(\theta).R_{ss}.a^H(\theta) + \sigma_b^2.I_M \qquad (6)$$

There have been numerous efforts to estimate DOA, encompassing methodologies like the multiple signal classification (MUSIC) and the rotational invariance technique referred to as ESPRIT. In this section, our focus is on DOA estimation through the ESPRIT technique, which capitalizes on the rotational invariance found within the signal subspace. This invariance emerges from utilizing two identical arrays with a structure featuring translational invariance. For instance, when examining an array with M antenna elements, it is composed of two distinct subarrays, neither of which overlap, each comprising (M-1) elements. Importantly, the elements within these subarrays exhibit matching radiation patterns [11] - [13]. The comprehensive received signal, encompassing contributions from both subarrays, is expressed as

$$\bar{x}(n) = \begin{bmatrix}\bar{x}_{ar1}(n)\\ \bar{x}_{ar2}(n)\end{bmatrix} = \begin{bmatrix}\bar{a}_1(\theta)\\ \bar{a}_2(\theta)\end{bmatrix}.\bar{s}(n) + \begin{bmatrix}\bar{b}_{ar1}(n)\\ \bar{b}_{ar2}(n)\end{bmatrix} \qquad (7)$$

Thus, $\bar{a}_2(\theta)$ is defined as $\bar{a}_1(\theta)\bar{\phi}$.

$$\begin{bmatrix}\bar{a}_1(\theta)\\ \bar{a}_2(\theta)\end{bmatrix} = \begin{bmatrix}\bar{a}_1(\theta)\\ \bar{a}_1(\theta)\bar{\phi}\end{bmatrix} \qquad (8)$$

At this point, we have the ability to compute either the input covariance or the covariance matrix R_{xx} for the array's output. Concurrently, the correlation matrices pertaining to the two subarrays are provided as:

$$\bar{R}_{xx} = E(\bar{x}.\bar{x}^H) = \bar{a}(\theta).\bar{R}_{ss}.\bar{a}^H(\theta) + \sigma_b^2.\bar{I} \qquad (9)$$

Compute the eigen decomposition of $\bar{R}_{xx}$, we get:

$$\bar{R}_{xx} = \bar{Q}\,\bar{\Lambda}_{xx}\bar{Q}^{\,H} \qquad (10)$$

Where

$\Lambda = \text{diag}\,[\lambda_1 , ... \lambda_{2M-2}\,]$ and $Q = \text{diag}\,[q_1 , ... q_{2M-2}\,]$ are the eigenvalues and eigen vectors respectively. Partition Q and decompose two signal subspace $\bar{E}_{s1}$ and $\bar{E}_{s2}$ respectively to obtain E_s,

$$\bar{E}_s = \begin{bmatrix} \bar{E}_{s1} \\ \bar{E}_{s2} \end{bmatrix} \quad (11)$$

Next form a 2L×2L matrix using signal subspaces, we can find the following expression

$$\overline{\mathrm{E_c}} = \begin{bmatrix} \overline{\mathrm{E}}_{s1}^{\mathrm{H}} \\ \overline{\mathrm{E}}_{s2}^{\mathrm{H}} \end{bmatrix} . [\overline{\mathrm{E}}_{s1} \overline{\mathrm{E}}_{s2}] = \overline{\mathrm{E}}.\overline{\mathrm{P}}.\overline{\mathrm{E}}^{\mathrm{H}} \quad (12)$$

And partition $\bar{E}$ into four L by L submatrices such that

$$\bar{E} = [\begin{matrix} \bar{V}_{11} & \bar{V}_{12} \\ \bar{V}_{21} & \bar{V}_{22} \end{matrix}] \quad (13)$$

The relation operator $\bar{\Psi}$ is given by

$$\bar{\Psi} = -\bar{V}_{12}.(\bar{V}_{22})^{-1} \quad (14)$$

Also it can be shown that:

$$\bar{E}_{s1}.\bar{\Psi} = \bar{E}_{s2} \quad (15)$$

It is necessary for a distinct and non-singular transformation matrix, denoted as $\bar{\mathrm{T}}$, to also be present in a manner such that

$$\bar{E}_{s1} = \bar{\mathrm{A}}.\bar{\mathrm{T}} \quad (16)$$

and

$$\bar{E}_{s2} = \bar{\mathrm{A}}.\bar{\phi}.\bar{\mathrm{T}} \quad (17)$$

Through the substitution of equations (14) and (15) into equation (17), the resultant expression is

$$\bar{\psi} = \bar{T}^{-1} * \bar{\phi} * \bar{T} \quad (18)$$

Hence, it is imperative for the eigenvalues of $\bar{\Psi}$ to match the diagonal components of $\bar{\Phi}$, ensuring that

$$\lambda_i = e^{-jksin(\theta_i)} \quad (19)$$

After finding the eigen values of $\bar{\Psi}$, the angles of arrival can be estimated as:

$$\theta_{\mathrm{i}} = sin^{-1}\left(\frac{\arg(\lambda_i)}{kd}\right), \text{ for } i = 1,2, \dots L \quad (20)$$

III. Simulation Results

A uniform linear array consisting of M=8 elements, spaced at intervals equivalent to half a wavelength, was utilized. Subsequently, a histogram was constructed to represent the DOA estimates, serving as a probability distribution for signal arrival directions. The histogram plots in Figure 3 portray azimuth DOA estimates for L=4 sources at angles of [30°, 50°, 70°, and 90°], employing the proposed technique. It is evident from the figure that our proposed algorithm provides a very close joint DOA estimation, with clear peaks appearing around [30°, 50°, 70° and 90°]. In the context of Figure 4, well-defined histograms exhibit azimuth DOA estimations using the proposed ESPRIT method. Here, six signals are present, emanating from DOAs of [30°, 50°, 70°, 90°, 120°, 140°], each with a signal-to-noise ratio (SNR) of 5 dB. The antenna array consists of eight elements. These figures demonstrate the effectiveness of the proposed effectiveness of the proposed algorithm in accurately estimating the azimuth DOA in different scenarios.

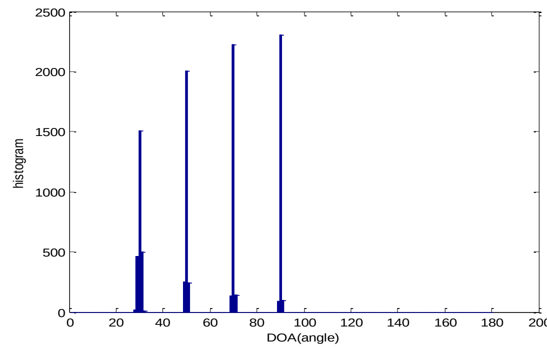

Fig. 3 Histogram bar chart of DOA estimation for L=4 sources of DOA [30°, 50°, 70°, 90°], SNR=5, and M= 8 elements

In this section, we delve into the outcomes of the ESPRIT algorithm simulation. Our focus commences with an examination of the case where three signals, arriving at angles of 30°, 60°, and 120°, are taken into consideration. Perturbations are introduced by varying the signal-to-noise ratio (SNR) and adjusting the array's element count. The observed relationship between the root mean square error (RMSE) and SNR, corresponding to different values of M (the number of array elements), is depicted in Figure 5. Notably, a conspicuous trend emerges: as the count of elements M diminishes, the magnitude of error becomes increasingly prominent. The same behavior was observed when using the MUSIC method. According to the RMSE values found, it is observed that the values found by the ESPRIT algorithm decrease as the SNR increases.

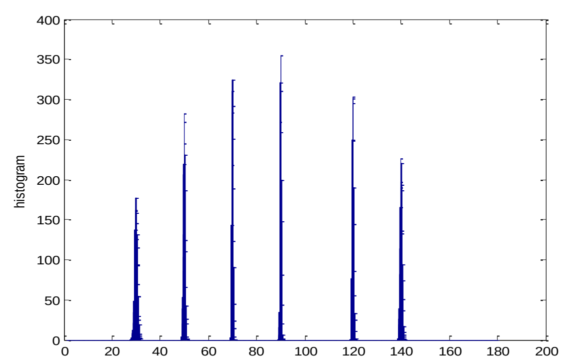

Fig. 4 Histogram bar chart of DOA estimation for L=6 sources of DOA [30°, 50°, 70°, 90° , 120°, 140°], SNR=5, and M= 8 elements

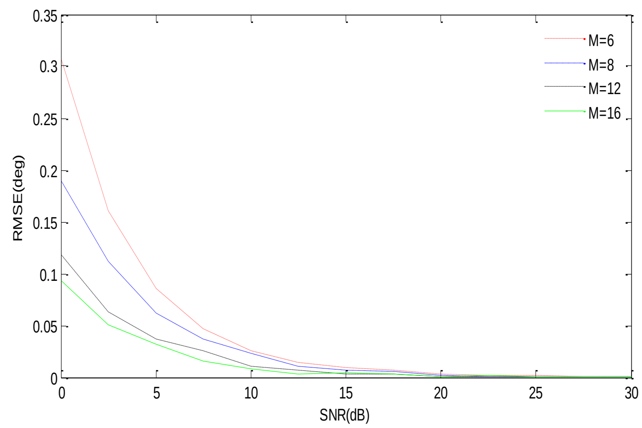

Fig. 5 Root mean square error (RMSE) of the DOA estimates versus the input SNR(L=3, θ_1=30°, θ_2=60°, θ_3=120°, N=2400, d=λ/2)

IV. Conclusion

This paper presents an investigation into the theory of smart antennas and the basic concepts of the ESPRIT algorithm. The performance of the ESPRIT algorithm is evaluated through computer simulations, which demonstrate the effect of various parameters on the algorithm's ability to accurately and efficiently resolve incoming signals. The results show that the performance of the ESPRIT algorithm can be improved by adjusting the following parameters: An increased number of elements within the array results in larger signal snapshot and a more significant angular separation between the signals. These enhancements manifest as heightened peak sharpness and a smaller error in angle detection. The ESPRIT algorithm effectively estimates the direction of arrival (DOA) for incoming on the antenna array, as indicated by the simulation outcomes.

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